OPTIMIZATION OF LINEAR CLOSED-LOOP SYSTEMS WITH APPLICATION TO TURBOJET ENGINE CONTROLS

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One of the fundamental functions of feedback controlled systems is to maintain certain variables, called errors, at constant or minimum values. This must be accomplished under the particular set conditions which include the dynamical nature of the process to be controlled, the difficulties in accurate measurements, servo limitations as to power and saturation, random disturbances and inputs, and noise. A major characteristic of the controller is its dynamic response, so designed as to minimize the errors under the restrictions mentioned.

In this paper one such design method is to be discussed. It starts with the specifications on and the limitations of the controlled system and leads directly to a unique and mathematically optimum dynamic characteristic of the control. The history of this approach started with Wiener's work (ref. 1) on the design of linear filters and predictors. Fig. 1 illustrates this open-loop control problem. Wiener's method leads to that linear structurally stable filter, or predictor, which minimizes the mean square error under random inputs.

Several researchers, in recent years, have recognized that this same theory can be applied to closed-loop control systems (refs. 2, 3, and 4). The key idea is to derive an equivalent open-loop system as shown on Fig. 2. The control is to be designed for the fixed servo and engine so as to minimize the error. The open loop shown on Fig. 2 places a filter between the input and the servo and is exactly equivalent dynamically to the closed-loop system; the relation between control and filter characteristics are shown on Fig. 2. The necessary and sufficient condition for stability of the closed loop system is that the filter be stable. This is exactly the only restriction placed on Wiener filter or predictor and the methods for that case can thus be applied. It should be noted that we can deal with either random or transient inputs. For random inputs spectral densities and cross spectral densities of the inputs are required. For transient inputs, where integral squares replace mean squares, products of Fourier transforms replace spectral densities.

Direct application of this method will sometimes give unrealistic answers, such as control gains approaching infinity or unrestricted derivative action. One cause of this is the unrestricted use of linearized characteristics far beyond their validity. Fig. 3 illustrates this problem for a component which saturates. Within the linear range, y is proportional to x. Outside the linear range, the apparent proportionality constant decreases as x or x^2 increases.

To account for this effect and still preserve the use of linear techniques, several methods have been suggested (refs. 3, 4, and 5). The simplest approach is to constrain the variation in y. There is a general equivalence between constraining the mean square variation in y and its natural constraint as shown on Fig. 3. For such a constraint, the filter theory method can easily be extended, as shown on Fig. 4. The closed-loop system has some variable, say y, which saturates or is to be limited. The equivalent open loop is the same as before with the variable y as an output along with the error. Now the filter is to be determined for $\overline{(\text{error})^2} + \lambda \overline{y^2}$ to be a minimum. The Lagrangian multiplier, λ, will appear as a parameter in the equation for the optimum filter and thus in the equation for the optimum control. Its value is adjusted for the allowable y^2. Then, for that particular value of y^2, the optimum control, having that corresponding value of λ, will give the minimum error^2.

In this paper, the applications are to turbojet engine controls. For these cases, constraints of a different kind exist. Even within the linear ranges of engine variables there is the possibility of engine damage or failure; for example, because of excessive temperature. The capabilities of the engine fuel system may be far beyond that required for over-temperature or combustion blow-out. In the examples to follow, constraints on temperature and fuel flow accelerations are used.

Another feature of turbojet engine controls is the necessity of manipulating several engine inputs (such as fuel flow, nozzle position, etc.). This multi-loop controlled system can be attacked by an extension of the method already presented. Fig. 5 shows such a general system. Matrices are used to represent the separate responses of each output of a component to each input. The dynamic characteristic represented by any one element in the matrix is that response of the output attached to its row to the input attached to its column.

The equivalent open-loop for the system of Fig. 5 is shown on Fig. 6. Again the filters are placed between the inputs and the fixed part of the system. For generality a separate matrix, H, and separate inputs, w, are used to generate the errors and constrained variables, e. The specification on the filter is the same as before where the λ's will then appear as parameters in the control and are used to set the values of the constrained variables. The relation between the F and C matrices are shown on Fig. 6. The necessary and sufficient condition for stability of the closed multi-loop system is that each element of the filter (F_{jk}) be stable.

In the examples to follow, the mathematical detail is omitted. The major assumptions are linearity and the use of mean squares and integral squares as the criteria on optimization. The limitations on the variation of certain variables because of their adverse effects or saturation is accounted for by constraining the mean square or integral square values of those variables. The principal analytical task in this method is the factoring of a real function of frequency, reference 1. In the multi-loop case, this problem becomes one of factoring a matrix, reference 2. It should be noted that the final controlled systems obtained are not only stable but structurally stable.

Example 1.

The first example, shown on Fig. 7, is the optimization of a speed control of a turbojet engine with a limitation on engine temperature. The idealized response of engine speed to fuel flow is shown as a lag with time constant, τ. The response of engine temperature to fuel flow is assumed just proportional. For the case of a transient step input in speed setting up to maximum speed, and a constraint on the integral square over-temperature, the optimum control has proportional plus integral action, where the integral time constant is equal to the engine time constant. The control gain G is the adjustable parameter of the control derived from the Lagrangian multiplier λ, and the effect of its adjustment is shown on Fig. 8. As the control gain is increased, the temperature error increases and speed error decreases. On this scale, a control gain of one gives zero temperature error and an integral square speed error of 8. The allowable over-temperature sets the proper control gain and the curve of Fig. 8 gives the minimum speed error obtained when the optimum control is used. This curve can also be used as a standard by which other non-optimum controls can be judged. The point determined by the integral square speed and temperature errors will lie above the curve and, if this point lies sufficiently close to the curve, the control cannot be much improved upon.

Example 2.

The second example, shown on Fig. 9, is also speed control of turbojet engine, this time with a limitation on fuel acceleration. For the case of a transient step input in speed setting and a constraint on integral square fuel acceleration, the optimum control has proportional plus integral action where again the integral time constant equals the engine time constant. An extra lag is added to this control, acting as the high frequency filter necessary to limit fuel acceleration. The control gain, G, and α are the adjustable parameters in the control derived from the Lagrangian multiplier, λ, and related as shown.

Fig. 10 shows that as the allowable fuel acceleration is increased, the proper control gain is increased, and α is decreased. The closed-loop response is that of an underdamped second order system having a damping ratio, ζ, almost constant at 0.7. The shallow slope (about 1:3) of the curve of Fig. 10 indicates the relative difficulty of decreasing the speed error by improving the fuel acceleration capabilities in the region shown.

Example 3.

The third example, shown on Fig. 11, is also speed control of a turbojet engine with a limitation on fuel acceleration. For the case of a transient step input in nozzle position, this time, and a constraint on integral square fuel acceleration, the optimum control has proportional plus integral action where, the integral time constant is not equal to the engine time constant. The control gain, G, and the integral time constant, σ, are the adjustable parameters in the control derived from the Lagrangian multiplier, λ, and related as shown.

Fig. 12 shows that as the allowable fuel acceleration is increased, the proper control gain is increased, and σ is decreased. The closed-loop response is that of an underdamped second order system and again the damping ratio, ζ, is almost constant at 0.7. The steep slope (about 3:1) of the curve of Fig. 12 indicates the relatively large advantage in improving the fuel acceleration capabilities in the region shown.

Example 4.

The fourth example, shown on Fig. 13, is the multi-loop case. It is desired to optimize the control of engine speed and temperature by manipulation of engine fuel flow and nozzle position. The idealized responses of engine speed to fuel flow and nozzle position are lags, as shown, and it is assumed that the response of temperature to fuel flow is just proportional and that temperature does not respond to nozzle position alone. The required control has four unknown characteristics; the responses of fuel flow and nozzle position to the speed error and temperature error signals. These error signals are assumed to have pure random noise added to the true errors.

For the case of independent transient step inputs in speed setting and temperature setting and pure random noise in the speed error pick-up and in the temperature error pick-up, the control, shown on Fig. 14, is the optimum.

We notice that both fuel flow and nozzle position respond to the temperature error signal with pure integral action. Fuel flow does not respond to the speed error signal. Nozzle position responds to the speed error signal with proportional plus integral action, where the integral time constant equals the engine time constant. This optimum control also results in noninteraction, whereby the speed error and temperature error signals do not affect each other.

The two control gains, G_1 and G_2, are the adjustable parameters derived from the Lagrangian multiplier, and the effects of their adjustment is shown by the equations on Fig. 14. As G_1 is increased, the transient speed error response to speed setting is decreased, whereas the mean square speed response to the noise is increased. As G_2 is increased, the transient temperature error response to temperature setting is decreased, whereas the mean square temperature response to the noise is increased. The gains, G_1 and G_2, are determined by a compromise between the effects of the transient inputs and the noise.

These examples have illustrated the application of the optimization procedure for the control of a turbojet engine. The mathematically optimum controls arrived at are quite realistic in practice and, for the examples chosen, are similar to those in use. It is expected that this method cannot only be used for design, but also as a standard by which controls can be evaluated, or as a possible basis for specifications on control dynamics.

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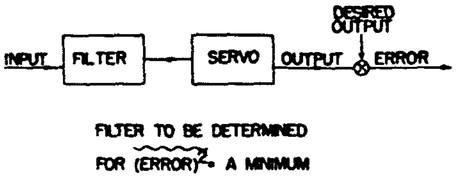


Fig. 1. - Block diagram of Wiener open-loop filter and predictor problem.

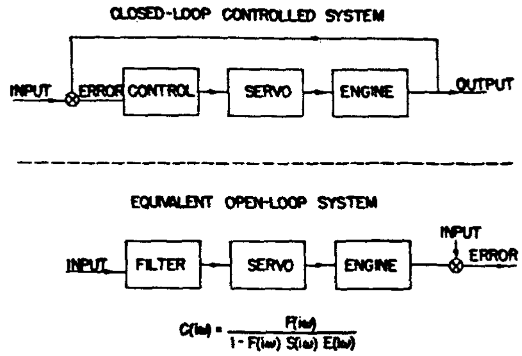


Fig. 2. - Equivalent open-loop system for application of filter theory to closed-loop control.

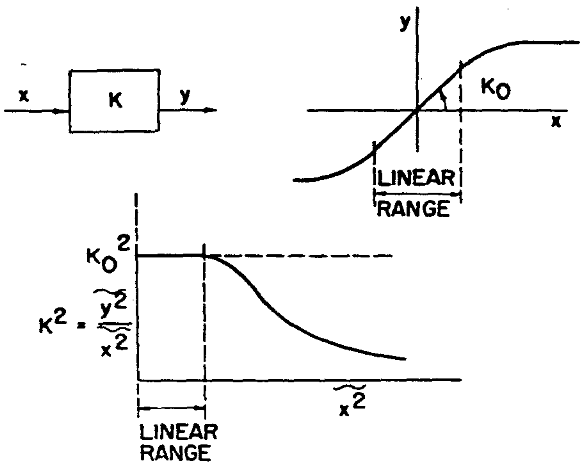


Fig. 3. - Effect of saturation on a linearized gain.

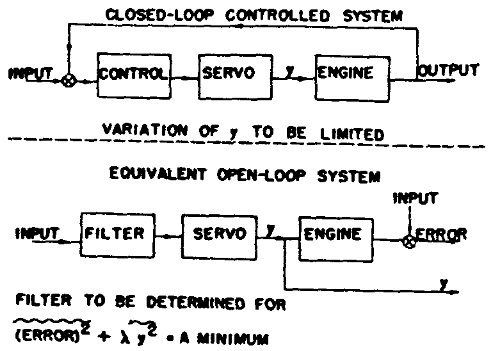


Fig. 4. - Use of open-loop filter theory with constraints or saturation.

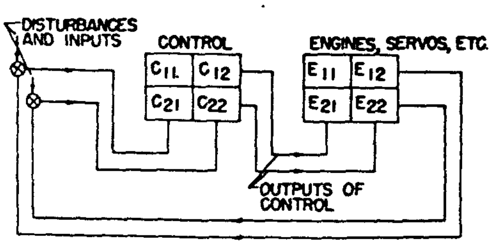
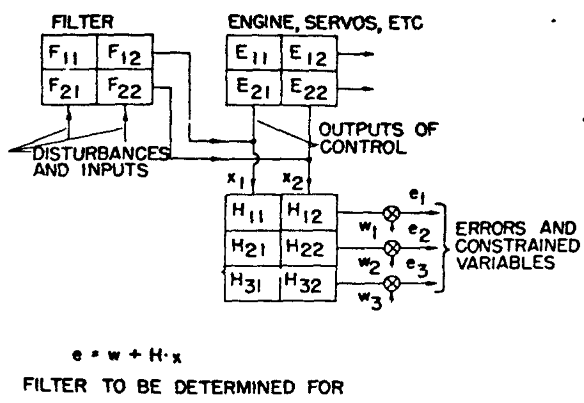


Fig. 5. - General closed-loop controlled system.



$$e = w + H \cdot x$$

FILTER TO BE DETERMINED FOR

$$\lambda_1 \overline{e_1^2} + \lambda_2 \overline{e_2^2} + \lambda_3 \overline{e_3^2} + \cdots = \text{A MINIMUM}$$

$$C = F(I - EF)^{-1}$$

Fig. 6. - Equivalent general open-loop system for application of filter theory to optimization of general closed-loop controlled system.

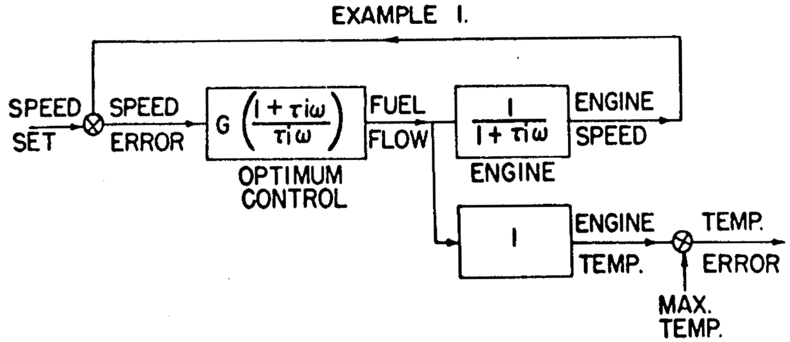


Fig. 7. - Optimization of speed control of turbojet engine with constraint on over-temperature under transient step inputs in speed setting.

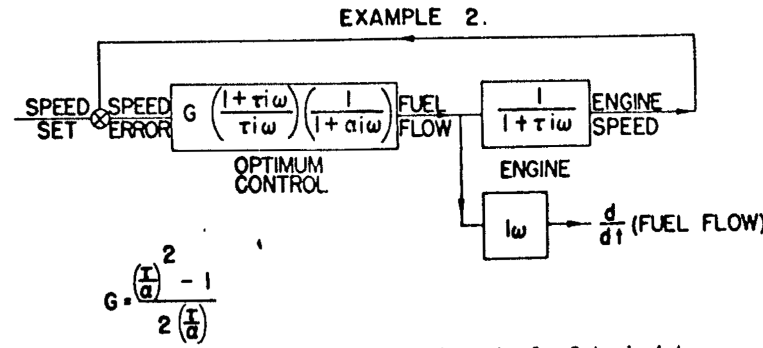


Fig. 9. - Optimization of speed control of turbojet engine with constraint on time derivative of fuel flow under transient step inputs in speed setting.

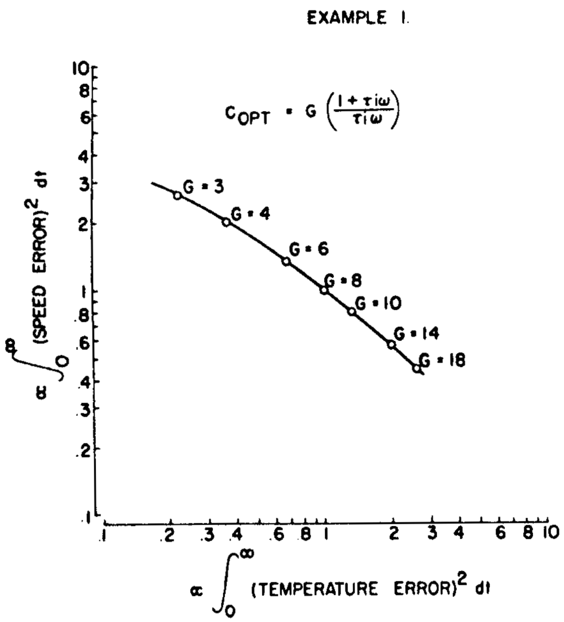


Fig. 8. - Effect of allowable over-temperature on minimum speed error for transient step inputs in speed setting.

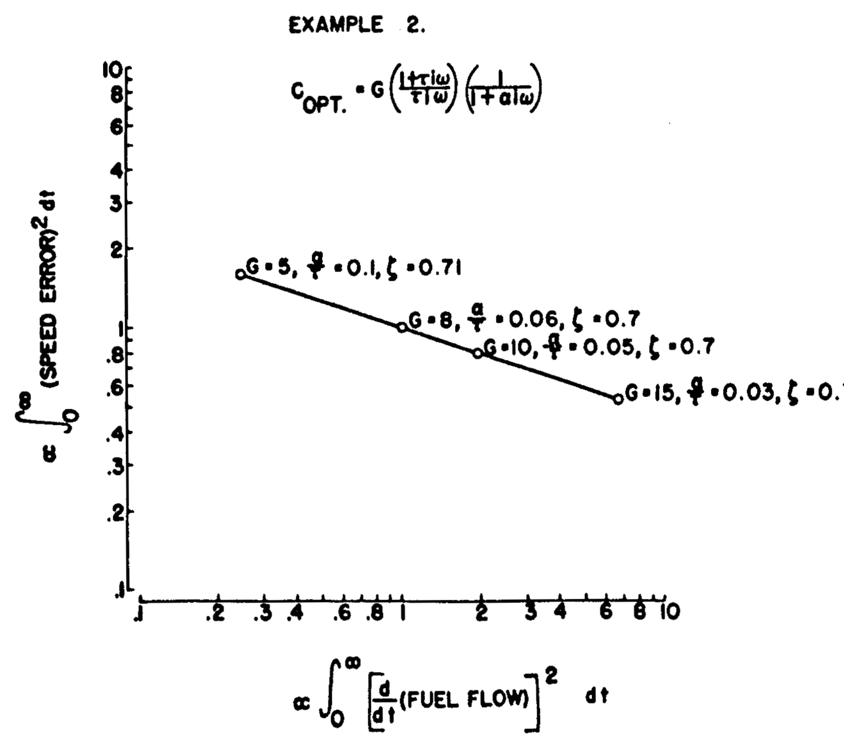
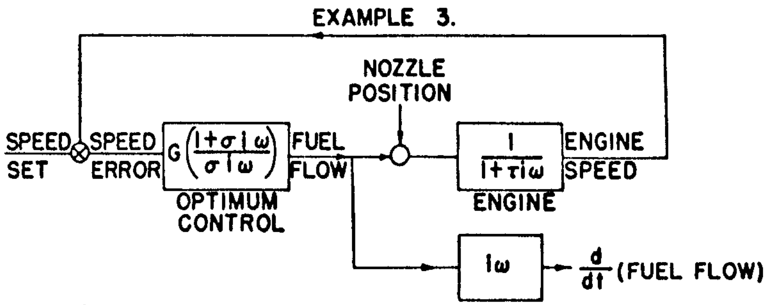


Fig. 10. - Effect of allowable fuel flow acceleration on minimum speed error for transient step inputs in speed setting.



$G = 2\left(\frac{\tau}{\sigma} - 1\right)$

Fig. 11. - Optimization of speed control of turbojet engine with constraint on time derivative of fuel flow under transient step inputs in nozzle position. Speed setting fixed.

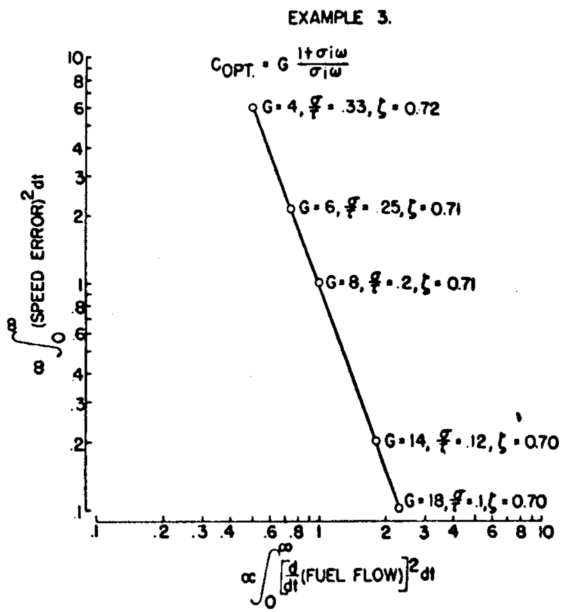


Fig. 12. - Effect of allowable fuel flow acceleration on minimum speed error for transient step inputs in nozzle position. Speed setting fixed.

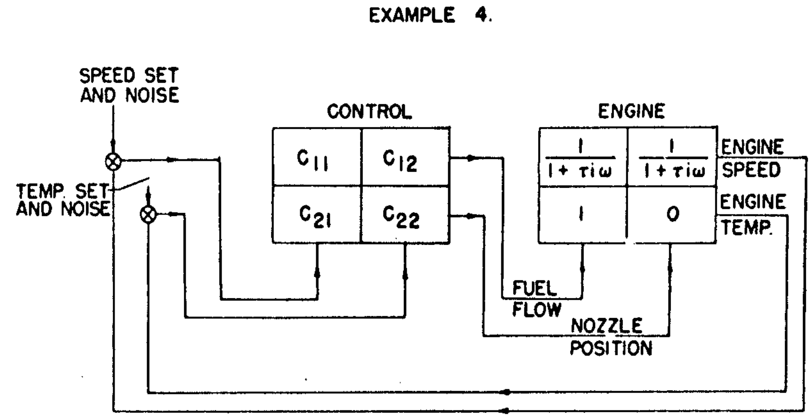
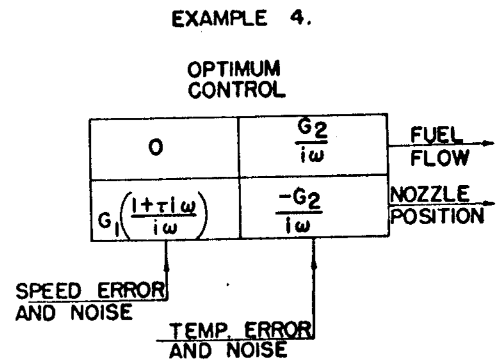


Fig. 13. - Speed and temperature control of turbojet engine with noise in speed and temperature instrumentation.



$$\int_0^\infty (\text{SPEED ERROR})^2\, dt \propto \frac{1}{G_1}; \quad \widetilde{(\text{SPEED ERROR})^2} \propto G_1$$

$$\int_0^\infty (\text{TEMP. ERROR})^2\, dt \propto \frac{1}{G_2}; \quad \widetilde{(\text{TEMP. ERROR})^2} \propto G_2$$

Fig. 14. - Optimization of speed and temperature control of turbojet engine under transient step inputs in speed setting and temperature setting and with pure random noise in speed and temperature instrumentation.